2.1 Product and Quotient Rules: Quiz D Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; provenance is in `PROVENANCE_QD.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Simplifying first, the product rule, and the quotient rule are all accepted wherever they apply; no method is required, and long but correct work gets full credit.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part unless a row says otherwise.
- **Domains.** On Q1, a correct note on where f' does not exist is accepted, never required.
- **Simplifying.** Answers need not be simplified beyond what a row says; equivalent forms are correct.

Quiz D

Item	Answer	Rubric / trap
1(a)	$\csc x - (x+2)\csc x \cot x$	product rule: $(1)(\csc x) + (x+2)(-\csc x \cot x)$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* multiplying the derivatives gives $-\csc x \cot x$; earns 0. The derivative of $\csc x$ carries a minus sign.
1(b)	$\dfrac{17}{(5-x)^2}$	quotient rule: $\dfrac{2(5-x)-(2x+7)(-1)}{(5-x)^2}$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* the derivative of $5-x$ is -1; using $+1$ gives $\dfrac{3-4x}{(5-x)^2}$.
1(c)	0 (for every x in the domain of f)	$\dfrac{(x-3)(x+3)}{2(x-3)(x+3)} = \frac{1}{2}$ wherever f is defined, a constant. The quotient rule, carried through, also gives 0. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant in disguise; the denominator is half factored, so the cancellation is easy to miss.
1(d)	$-2\csc x \cot x$	$\cos x \tan x = \sin x$, so $\dfrac{\sin x}{\cos x \tan x} = 1$ wherever it is defined and $f(x) = 2\csc x - 1$. The quotient rule also gives 0 for that term: its numerator is $\cos x \cdot \sin x - \sin x \cdot \cos x$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant hidden in a quotient; the derivative of $\csc x$ is $-\csc x \cot x$, with a minus sign.
1(e)	0	π and $\csc\left(\frac{\pi}{6}\right) = 2$ are numbers, so $f(x) = 6\pi$ for every x. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* no x appears; 6 (treating π as the variable) or any product-rule result earns 0.
2	$y - 1 = x - \frac{\pi}{2}$, i.e. $y = x + 1 - \frac{\pi}{2}$	1 pt $f\left(\frac{\pi}{2}\right) = \frac{1}{1+0} = 1$; 2 pts $f'(x) = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$; 1 pt $f'\left(\frac{\pi}{2}\right) = \dfrac{0+1}{1} = 1$; 1 pt equation, any correct form. *Trap:* the derivative of $1+\cos x$ is $-\sin x$, so the second term becomes $+\sin^2 x$; dropping that minus gives slope -1.

Item	Answer	Rubric / trap
3	$x = \frac{5\pi}{6}$ and $x = \frac{3\pi}{2}$	1 pt $f'(x) = -\sin x(1 + \sin x) + \cos^2 x$; 2 pts $\cos^2 x = 1 - \sin^2 x$: $2\sin^2 x + \sin x - 1 = 0$, $(2\sin x - 1)(\sin x + 1) = 0$; 2 pts both solutions in the interval and nothing else. *Trap:* dropping the factor $\sin x + 1$ loses $\frac{3\pi}{2}$ (the tangent there is horizontal even though f has no maximum or minimum there); $\frac{\pi}{6}$ is outside the interval.
4(a)	5	Readings: $f(1) = 2$, $f'(1) = -1$, $g(1) = -1$, $g'(1) = -2$ (slopes are rise over run of the segment containing $x = 1$). $q'(1) = \dfrac{f'(1)g(1) - f(1)g'(1)}{[g(1)]^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(1)/g'(1) = \frac{1}{2}$; reversed numerator gives -5; swapping the solid and dashed graphs gives $-\frac{5}{4}$; reading a height where a slope is needed.
4(b)	3	Readings: $f(3) = -1$, $f'(3) = -2$, $g(3) = -2$, $g'(3) = 1$ (slopes are rise over run of the segment containing $x = 3$). $p'(3) = f'(3)g(3) + f(3)g'(3)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(3)g'(3) = -2$; $f(3)g(3) = 2$; mixing a value from one graph with a slope from the other.
5(a)	6	$k'(x) = \dfrac{2x\,g(x) - x^2 g'(x)}{[g(x)]^2}$, $-\frac{1}{2} = \dfrac{2(2)(4) - 4g'(2)}{16}$; substitute the given values and solve for $g'(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* the item gives the derivative of the quotient and asks for a factor's derivative; reversing the numerator gives 2.
5(b)	3	$p'(x) = f'(x)g(x) + f(x)g'(x)$, $10 = 4\,g(3) + (-2)(1)$; substitute the given values and solve for $g(3)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* this part asks for a function value, $g(3)$; $p' = f'g'$ gives $4 = 10$, a contradiction, not an answer.
6(a)	$\frac{7}{8}$	$h'(2) = \dfrac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2}$, with the row $x = 2$. 1 pt rule, 1 pt entries, 1 pt value. *Trap:* reversing the numerator gives $-\frac{7}{8}$; $f'(2)/g'(2) = 3$; the g/f version gives $-\frac{7}{2}$. Columns run x, $g(x)$, $f'(x)$, $g'(x)$, $f(x)$.
6(b)	2	$k'(x) = g(0)$, with the row $x = 4$. 1 pt, all or nothing. *Trap:* $g(0) = 2$ is a number, but k is that number times x, not a constant: answering 0 earns 0. Reading the $x = 4$ row also earns 0.

Points: $10 + 5 + 5 + 8 + 8 + 4 = 40$.